REVIEWS

Surveys in Mechanics: G. I. Taylor 70th Anniversary Volume, edited by G. K. BATCHELOR and R. M. DAVIES. Cambridge University Press, 1956. 475 pp. 50*s*. or $9.50.

It would be difficult to imagine a happier birthday present than a dedication volume of papers. The notion is not new, but few such volumes can have been so personally suited as this one is to Sir Geoffrey Taylor. Far from being an assortment of research progress reports by fellow scientists wishing to pay their respects, this is a limited collection of surveys of "fields of knowledge which he has made his own".

Following a biographical note by Sir Richard Southwell, the contents are as follows.

The mechanics of quasi-static plastic deformation in metals,	by R. Hill
Dislocations in crystalline solids,	by N. F. Mott
Stress waves in solids,	by R. M. Davies
Rotating fluids,	by H. B. Squire
The mechanics of drops and bubbles,	by W. R. Lane and H. L. Green
Wave generation by wind,	by F. Ursell
Viscosity effects in sound waves of finite amplitude,	by M. J. Lighthill
Turbulent diffusion,	by G. K. Batchelor and A. A. Townsend
Atmospheric turbulence,	by T. H. Ellison
The mechanics of sailing ships and yachts,	by K. S. M. Davidson

Readers of this Journal will not need to be told that this set of topics falls short of those to which G. I. Taylor has made central contributions. Isotropic turbulence dynamics (in fact, the very notion of isotropic turbulence), turbulent shear flow, propulsion at very small Reynolds numbers, flow past porous boundaries—these too could have been included. Like other fluid dynamicists, I have more than once had the experience of finding a pet idea stolen by G. I. Taylor decades before I even thought of it!

The singular talents of G. I. Taylor as a researcher need no more testimony than his own published—and unpublished—works. In an age of increasing dichotomy between theoretical and experimental research, a teacher can point to no better illustration of the benefits to be gained from combining the two.

The papers in this volume are of good quality and some are outstanding, though the group is variable in spirit and in purpose. My own limitations preclude pertinent comment on the solid mechanics work; perhaps a pardonable omission in this review. No single survey here is completely reportorial or completely critical, although the Lighthill, Ursell and Batchelor–Townsend contributions come closest to the latter.

Before taking up the individual fluid mechanics papers, I can't resist observing that Sir Geoffrey's direct descent from the mathematician Boole (mentioned in the pleasant biographical sketch by Sir Richard Southwell) in a sense runs counter to the old adage that scientific talent is handed down from father to son-in-law.

Rotating fluids

This is a clear, well-organized summary of work on disturbances in fluids which are basically in rigid body rotation. The chief headings are "1. Small disturbances", "2. A point source on the axis of rotation" and "3. Steady symmetric motions."

Part 2, apparently an improved version of a problem of Barua (*Quart. J. Mech. Appl. Math.* **8**, 1955), still has an *ad hoc* flavour. No criticism of the original paper is offered. Part 3, which includes the non-linear aspects, may treat them a bit more briefly than some specialists might prefer, but it does include some stimulating discussion.

Perhaps in the interest of brevity, the title seems more general than necessary; for example, there is much additional work on rotating fluids in the meteorological literature.

The mechanics of drops and bubbles

The topics treated here are: a particular kind of pendant drop, methods of producing drops, the behaviour of falling drops and their break-up, deposition of drops on obstacles, production of bubbles, and the dynamical behaviour of bubbles under various conditions.

This is a good job of assembling and organizing some remarkably dispersed groups of papers. As many readers will realize, the research literature in some of these areas is not only spread throughout a great assortment of professional journals, but is also presented from divergent points of view.

Since much of the work has been done with practical motivations, the publications often emphasize semi-empirical 'correlations' among parameters rather than a deductive approach based on fundamental fluid dynamics. In a sense these authors have continued the tradition by restricting themselves to reporting past work without pointing out the places where adverse criticism might be appropriate. Typical of this approach is the presentation of small Reynolds number droplet dynamics in impingement problems. Here the equations of motion are simply set down with the quasi-steady Stokes drag for a sphere already inserted. There is no hint of the considerable machinery of restriction and approximation which must be invoked.

Wave generation by wind

This is a gratifyingly critical article on a branch of hydrodynamics which seems on the verge of blossoming. The status of most of the older theoretical works is summarized in the following sentence. "Some hypotheses have thus remained without proof or disproof for many years, and through lapse of time are now taken more seriously than their authors would have wished." Quite apart from its merit as a piece of diplomacy, this sentence refers to a phenomenon noticeable elsewhere. Many engineers of the last 20 or 30 years have had far more literal faith in the 'mixing length' than did Taylor or Prandtl.

The general topics covered are as follows. Simple theory of stability of an interface, semi-empirical theories of wave generation, semi-empirical systems of wave forecasting, sea roughness, and surface slope due to wind.

There is an occasional lack of explicitness in describing experimental arrangements, especially noticeable where the traditionally aerodynamic concept of 'boundary layer' is involved.

The bibliography seems reasonable; perhaps the best known paper omitted is the extensive treatise of Hellström (*Ing. Vegen. Akad. Proc.* **158**, 1941).

Perhaps the most satisfying aspect of this paper is the emphasis on work that should be undertaken. It is not surprising to learn that well-defined laboratory experiments are given the highest priority. Research appears to be at the stage of evolution at which simplified laboratory studies aimed at an understanding of the basic mechanisms of phenomena are often dismissed as 'unrealistic' by professional oceanographers. This is presumably characteristic of any activity passing from a descripto-empirical stage to a scientific stage. In schools of oceanography in the United States, for example, it seems generally expected that even theoreticians should get their feet wet. How long is it since theoretical aerodynamicists were expected to hold pilot licences?

Atmospheric turbulence

Closely related to the problem of wind generated water waves is that of atmospheric turbulence. The general topics covered in this account are: mean profiles of wind, temperature and water vapour near the ground, experimental work on mean profiles, and turbulent fluctuations.

This is a generally good presentation, and is oriented more from meteorology than from fluid dynamics. The comparison of independent sets of data gives a critical aspect to the account. On the other hand, the invocation of traditional semi-empirical formulae is largely uncritical. For example, in discussion of the diffusion equation often used for heat and water vapour transport studies, Dr Ellison choses to omit mention of the apparent invalidity of the simple gradient transport concept here. A necessary condition for its applicability is that the characteristic convective scale (mean free path in kinetic theory, Lagrangian length scale here) be sufficiently small relative to the mean concentration field, and it is well known that this condition is not satisfied by conventional turbulent flows. Yet gradient transport calculations can often 'correlate' empirical data quite well, so we might say that this approach is not useful in principle, but merely in fact.

Although meteorologists seem more appreciative of laboratory experiments than do oceanographers, limitations of space apparently precluded coverage of the few but pertinent laboratory results on turbulent boundary layers over rough walls.

The mechanics of sailing ships and yachts

The first impression of this paper is that it has been written partly in some language other than English. Bewildered by the use of salty argot at crucial points in the exposition, I fortunately had a little-thumbed copy of *Shanty Men and Shanty Boys* by W. M. Doerflinger (New York: Macmillan, 1951). Having decoded the message, I found the story both fascinating and astonishing—fascinating because of what has been accomplished and recorded in the art of designing and operating sailing ships, astonishing because so little use has been made up to the present time of scientific progress in fluid mechanics.

The latter fact suggests that the sailing ship is on the way out. Or perhaps the yacht racing set is less wealthy than is commonly believed. On the other hand, it is possible that the sea is a force for conservatism; there is no great evidence that design of motor driven ships has taken much advantage of modern fluid dynamics either.

In reference to the technical content, it might have been more appropriate to title this paper "the *performance* of sailing ships and yachts". The problem of sailing against the wind is very well expounded. On the other hand there is a lack of dynamical explanation for the speed figures tabulated. In general one gets the impression of an intriguing field of engineering in which the most exciting problems remain to be solved.

Turbulent diffusion

This is a general and authoritative survey of the field of turbulent dispersion, with particular emphasis on the authors' own contributions. Since this domain is somewhat closer to my line of work than the rest of the volume, I take the liberty of more explicit comments*.

This exposition has such good qualities of lucidity and logical organization that newcomers to the subject may get the impression of a more satisfactory current status for the turbulent diffusion problem than actually exists. A possible reason is some tendency toward de-emphasis of the distinctions among empirical facts, intuitive conjectures, and theoretical results.

Typical illustrations are as follows. (*a*) The authors point out (p. 356) that the transition probability Q, "if it is like other probability density functions relating to turbulent motion . . . , will not be too far from the normal or Gaussian form." Nothing in this text vicinity warns the reader that the Gaussian form of these other p.d.f.'s is a partly empirical fact and, furthermore, that very few kinds have ever been measured. (*b*) On pp. 358, 359, the unwary neophyte might get the impression that both particle displacement probability P and transition probability Q have been shown to satisfy classical diffusion equations. Of course, this is not a deductive result —as the authors point out on p. 360. (*c*) The reader is not warned that the method used to present the data on diffusion in decaying isotropic turbulence, although successful for existing data, is restrictive: it requires the Eulerian and Lagrangian lengths to be simply proportional during decay. Although this may be a good approximation during the 'initial period' of decay, the authors certainly would not wish to convey this as a general property of turbulence. (*d*) Reference on p. 392 to later "deductions" that the average product of the three principal strain rates is negative may imply to the literate reader that this is a purely theoretical result. In fact, it is partly empirical as of the publication date of the volume.

Another section for cautious reading is that on fluid line and surface stretching. The presentation here is as heuristic as the pioneering paper of Dr Batchelor. Having encountered a number of fluid dynamicists with the impression that the original paper *proved* that fluid lines stretch,

* S. Potter, *The Art of Reviewsmanship*, New York Times, 8 May, 1955.

I should like to observe that the authors make no such claim. The fluid line stretching is a conjecture (and inferentially an empirical result from vorticity behaviour). The exponential rate follows from quite intuitive assumptions.

The discussion of lateral diffusion in shear flow is a mixture of careful formalism and *ad hoc* simplifications. The derivation of equations through (3.19) is an improvement over earlier published work. Then the quasi-Gaussian postulate is invoked in a situation where the non-Gaussian character of the fluctuation field may well be crucial. With a bit of hand waving, it is likely that the consequent approximation, $\overline{u'v^3} \doteqdot 3\overline{u'v}\,.\,\overline{v^2}$, can be qualitatively justified : $\overline{v^2}$ is positive, and v^3 has the same sign as v. Whether it can be given any quantitative justification seems unclear. Somewhat more startling is the bland assumption that $\overline{u'v^2} \doteqdot \overline{u'v}\,.\,\overline{v^3}/\overline{v^2}$. It is not evident *a priori* that even the sign of this approximation is correct.

The brief discussion of the behaviour of a diffusive scalar field embedded in an isotropic turbulence ignores much of the existing work on the subject (Obukhov, *Izvest. Akad. Nauk, S.S.S.R., Ser. Geofiz.* **13**, 1949, and Corrsin, *J. Aero. Sci.* **18**, 1951; *J. Appl. Phys.* **27**, 1951).

There are at least two specific improvements over previous writings on these subjects by Drs Batchelor and Townsend. (A) Equation (5.1), expressing the rate of growth of a fluid surface element, is in more elegant form than in the original paper of Batchelor. (B) The turbulence behind a regular grid is no longer referred to as being isotropic. The non-isotropy of these flows has been known to turbulence researchers in the United States since perhaps 1942 (Corrsin, A.E. Thesis, Cal. Tech., 1942), but the great importance and volume of the published grid turbulence data coming from the Cavendish Laboratory has given a rather general impression that virtual isotropy prevails within 16 or 20 mesh length of the grid. The latest Cavendish data are apparently in better agreement with those that have been taken at other laboratories.

From a pedagogical viewpoint the only serious omission is perhaps a statement of the nature of the problem of relating Lagrangian dispersion statistics to Eulerian statistical information. This, after all, is a possible definition of 'the problem' of turbulent diffusion. Given the statistical properties of the Eulerian (space-time) velocity field $u_i(x_j, t)$, the Lagrangian velocity field is

$$v_i(\mathbf{a}, t) = u_i(\mathbf{X}(\mathbf{a}, t), t). \tag{1}$$

The fluid point displacement is

$$X_k(\mathbf{a}, t) = \int_0^t v_k(\mathbf{a}, t)\, dt, \tag{2}$$

and

$$a_j = X_j(\mathbf{a}, 0).$$

Each realization of the Lagrangian time variation following a tagged fluid point (a_i = constant) is thus the variation along a trajectory through a member of the ensemble of Eulerian fields. The central difficulty of the problem is indicated by noting that each irregular 'sampling trajectory' is analytically related to the field it samples. Therefore we must deal with a stochastic integral equation. Putting (1) in (2) gives

$$X_k(\mathbf{a}, t) = \int_0^t u_k(\mathbf{X}(\mathbf{a}, t_1), t_1)\, dt_1. \tag{3}$$

The Lagrangian two-point correlation tensor is

$$L_{ik} \equiv \overline{v(\mathbf{a}, t)v_k(\mathbf{a}+\boldsymbol{\alpha}, t+\tau)}$$
$$= \overline{u_i(\mathbf{X}(\mathbf{a}, t), t)u_k(\mathbf{X}(\mathbf{a}+\boldsymbol{\alpha}, t+\tau), t+\tau)}.$$

For stationary fields, this depends on $(\boldsymbol{\alpha}, \tau)$ only. Taylor expressed the mean square particle displacement in terms of the Lagrangian time correlations $L_{11}(0, \tau)$.

The theory of correlation functions obtained by sampling with random intervals over an ensemble of functions appears to be new to probability theory, even for the degenerate case in which the individual intervals are unrelated to the individual members of the ensemble being sampled. Yet we face here the still more complicated case in which each member of the function ensemble determines its own sampling interval through the integral operation (3).

Viscosity effects in sound waves of finite amplitude

This is a condensed 100 page review which treats virtually all phases of the title subject. Especially noteworthy are the careful clarification and correction of the various traditional approximations, and the use of Burgers' equation (where it is really appropriate), designed originally as a 'model' of turbulence. Equally appealing is the author's consistent emphasis on the Reynolds numbers of the various kinds of waves. The principal subject headings are as follows. The physical mechanism of viscosity and other diffusion effects in gases, attenuation of sound due to relaxation effects, classical theory of shock wave formation, structure of stronger shock waves, more general theory of plane shock wave formation, interiors of unsteady shock waves, formation and decay of non-plane shock waves, sound waves of moderate Reynolds number, relaxation effects in sound waves of finite amplitudes.

The remarkable coverage of this review will undoubtedly make it a standard reference for a long time to come. There are perhaps two other facets that could be included to round out the subject. Chapter two, on the physical mechanism, stops short of statistical mechanics. At the phenomenological level of the fluid dynamicist, the underlying physical mechanism resides in the molecular dynamics. Professor Lighthill has chosen to omit discussion of the challenging and poorly understood problem of how the probability density of molecular velocities changes from one form to another in a few mean free paths through a shock wave. It might have been appropriate to make special mention of the measurements of F. S. Sherman, giving profiles through a shock wave (*Nat. Adv. Comm. Aero., Wash., Tech. Note* no. 3298, 1955).

In conclusion, this volume is an impressive tribute to one of the great engineering scientists of this century. The only Taylor-associated writings I expect to enjoy more are the research papers he will publish himself in the future!

S. Corrsin